Math (P)refresher for Political Scientists

4-14 September 2007

Breakfast 9am - 9:30am
Lecture 9:30am - 12:00pm
Section 1:00pm - 4:00pm

Ellie Powell (Instructor)
enpowell@fas.harvard.edu

Jen Katkin (Teaching Fellow)
jkatkin@fas.harvard.edu

Gary King (Faculty Adviser)
king@harvard.edu

Mailing list
math-pre@latte.harvard.edu

PURPOSE: Not only do the quantitative and formal modeling courses at Harvard require mathematics and computer programming — it's becoming increasingly difficult to take courses in political economy, American politics, comparative politics, or international relations without encountering game-theoretic models or statistical analyses. One need only flip through the latest issues of the top political science journals to see that mathematics have entered the mainstream of political science. Even political philosophy has been influenced by mathematical thinking. Unfortunately, most undergraduate political science programs have not kept up with this trend — and first-year graduate students often find themselves lacking in basic technical skills. This course is not intended to be an introduction to game theory or quantitative methods. Rather, it introduces basic mathematics and computer skills needed for quantitative and formal modeling courses offered at Harvard.

PREREQUISITES: None. Students for whom the topics in this syllabus are completely foreign should not be scared off. They have the perfect background for this course — the ones in most need of a "prefresh"ing before they take further courses with technical content. Students who have previously had some of this material, but have not used it in a while, should take this course to "refresh" their knowledge of the topics.

STRUCTURE & REQUIREMENTS: The class will meet twice a day, 9:00am – 12:00pm and 1pm – 4:00pm. This course is not for credit and has no exams. No one but the student will know how well he or she did. However, it still requires a significant commitment from students. Students are expected to do the reading assignments before the classes. Lectures will focus on major mathematical topics that are used in statistical and formal modeling in political science. Sections will be divided into two parts. During problem-solving sections, students are given exercises to work on (or as homework if not finished then), which should be handed in the following day. Students are encouraged to work on the exercises in groups of two or three. You learn more quickly when everyone else is working on the same problems! The exercises will be checked for errors to give students an indication of how well they are assimilating the material. During computing sections, we will introduce you to the computing

environment and software packages that are used in the departmental methods sequence. Math isn't a spectator sport — you have to do it to learn it.

COMPUTING: All of the methods courses in the department, and increasingly courses in formal theory as well, make extensive use of the computational resources available at Harvard. During the prefresher, students will be introduced to Harvard's Unix-based computing environment. Access to these servers is through VNC sessions, which enable students to work from any computer. Students will be introduced to Xemacs (text editing software), Latex (a typesetting language useful for producing documents with mathematical content), and R (the statistical computing language/environment used in the department's method courses). These resources are very powerful, but have something of a steep learning curve; one of the goals of the prefresher is to give students a head start on these programs.

TEXTBOOKS: The required text for this course is the textbook by Jeff Gill. The course will use the second printing of the text, which will be available later this summer. *Please wait to purchase the text until the second printing becomes available.*

1. Gill, Jeff. 2006. *Essential Mathematics for Political and Social Research.* Cambridge, England: Cambridge University Press.

There are several optional/recommended texts that you may wish to consult during the course. These texts will be available on reserve, but you may want to purchase some of them for your own future reference. In particular, Simon and Blume is useful for those who will be taking formal modeling courses in the Government or Economics departments.

2. Simon, Carl P. and Lawrence Blume. 1994. *Mathematics for Economists.* New York: Norton.

3. Wackerly, Dennis, William Mendenhall, and Richard Scheaffer. 1996. *Mathematical Statistics with Applications,* 5th edition.

4. Hahn, Harley. 1996. *Harley Hahn's Student Guide to Unix,* 2nd edition.

Lecture Schedule:

- **Lecture 1: Introduction - Notation and Functions (Tuesday, 4 September; CGIS Knafel N354)**

Topics:	R1 and Rn. Interval Notation for R1. Neighborhoods. Open/Closed/Compact Sets. Introduction to Functions. Domain and Range. Some General Types of Functions. Log, Ln, and e. Solving for Variables. Finding Roots. Limits of Functions. Continuity.
Required Reading:	Gill Ch. 1, 5.2
Further Reading:	SB 2.1-2, 12.3-5, 10.1, 13.1-2, 5.1-4

- **Lecture 2: Calculus I (Wednesday, 5 September; CGIS Knafel N354)**

Topics:	Sequences. Limit of a Sequence. Derivatives. Higher-Order Derivatives. Maxima and Minima. Composite Functions. The Chain Rule. Derivatives of Exp and Ln. L'Hospital's Rule.
Required Reading:	Gill Ch. 5.3-4, 6.4
Further Reading:	SB 12.1-2, 2.3-6, 3.1-2, 3.5, 4.1-2, 5.5

- **Lecture 3: Probability I (Thursday, 6 September; CGIS Knafel N354)**

Topics:	Counting. Sets. Probability. Conditional Probability and Bayes' Rule. Independence.
Required Reading:	Gill Ch. 7
Further Reading:	WMS 2.1-11

- **Lecture 4: Probability II (Friday, 7 September; CGIS Knafel N354)**

Topics:	Levels of Measurement. Discrete Distributions. Continuous Distributions. Joint Distributions. Expectation. Special Discrete Distributions. Special Continuous Distributions. Summarizing Observed Data.
Required Reading:	Gill Ch. 8
Further Reading:	WMS 3.1-4, 3.8, 4.1-5, 4.8

- **Lecture 5: Linear Algebra I (Monday, 10 September; CGIS Knafel N354)**

Topics:	Working with Vectors. Linear Independence. Matrix Algebra. Systems of Linear Equations. Method of Substitution. Gaussian Elimination. Gauss-Jordan Elimination.
Required Reading:	Gill Ch. 3
Further Reading:	SB 10.1-4, 11.1, 8.1-3. 6.1, 7.1

- **Lecture 6: Linear Algebra II (Tuesday, 11 September; CGIS Knafel N354)**

Topics:	Matrix Methods. Rank. Existence of Solutions. Inverse of a Matrix. Linear Systems and Inverses. Determinants. Determinant Formula for an Inverse. Cramer's Rule.
Required Reading:	Gill Ch. 4
Further Reading:	SB 7.2-4, 8.4, 9.1-2, WG Appendix A

- **Lecture 7: Introduction to Statistics (Wednesday, 12 September; CGIS Knafel N354)**

Topics:	Properties of Variance and Covariance. Conditional Variance. Properties of the Normal Distribution. Inference. Confidence Intervals.
Required Reading:	TBA

- **Lecture 8: Calculus II (Thursday, 13 September; CGIS Knafel N354)**

Topics:	Partial Derivatives The Indefinite Integral: The Antiderivative. The Definite Integral: The Area under the Curve. Integration by Substitution. Integration by Parts.
Required Reading:	Gill Ch. 6.2-3, 5.5-6
Further Reading:	SB 14.1, 14.3-4, Appendix 4.1-3

- **Lecture 9: Optimization 1 and Optimization 2 (Friday, 14 September; CGIS Knafel N354)**

  Optimization 1

Topics:	Quadratic Forms. Definiteness of Quadratic Forms. Maxima and Minima in Rn. First Order Conditions. Second Order Conditions. Global Maxima and Minima.
Required Reading:	Gill Ch. 4.9, 6.7
Further Reading:	SB 16.1-2, 17.1-4

  Optimization 2

Topics:	Constrained Optimization. Equality Constraints. Matrix Representation. Inequality Constraints: Kuhn-Tucker Conditions. Methods of Proof. Direct Proof. Proof by Contradiction. Proof by Induction.
Required Reading:	Gill Ch. 6.8
Further Reading:	SB 18.1-6, A1.3