Government 2332

Lecture 10

Regulating and Auditing
Natural Monopoly II

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Baron-Besanko Model

Motivations:

1. Regulators do audit
2. Even with mechanism design, addition of auditing may make for greater welfare
3. [Not an explicit motivation] Might regulators over- or under-regulate, or err?

Preliminaries

Firm has total cost function as sum of fixed plus variable cost. Fixed is common knowledge, variable is private information.

$$\tilde{C} = \tilde{c}Q + k \quad (1)$$

$G\left(c|\theta\right)$ is conditional distribution of marginal cost

$$G\left(c|\theta_1\right) \quad FOSD \quad G\left(c|\theta_2\right), \forall\theta_1 > \theta_2, s.t. E\left[G\left(c|\theta_2\right)\right] \leq E\left[G\left(c|\theta_1\right)\right]$$

Welfare Function

Expected penalty (payoff of audit), given theta, is

likelihood function

$$E\left[N\left(\tilde{\theta},\tilde{C}\right)|\theta\right] = \int_{\Gamma} N\left(\tilde{\theta},C\right) h\left(C|\theta\right)dC$$

support of C

Total welfare is

$$W = \int_{\theta_0}^{\theta_1} \left\{V\left(Q\left(p\left(\theta\right)\right)\right) - p\left(\theta\right)Q\left(p\left(\theta\right)\right) - s\left(\theta\right) - a\underline{\beta\left(\theta\right)} + \beta\left(\theta\right)E\left[N\left(\theta,\tilde{C}\right)|\theta\right] + \alpha\pi\left(\theta\right)\right\} f\left(\theta\right)d\theta \tag{5}$$

subsidy/transfer

Audit cost

Audit probability (eqm)

profit

Welfare and State Equations

$$W = \int_{\theta_0}^{\theta_1} \left\{ V\left(Q\left(p\left(\theta\right)\right)\right) - \tilde{c}\left(\theta\right) Q\left(p\left(\hat{\theta}\right)\right) - k - a\beta\left(\theta\right) - \left(1-\alpha\right)\pi\left(\theta\right) \right\} f\left(\theta\right) d\theta \tag{8}$$

$$\frac{d\pi\left(\theta\right)}{d\theta} = -\tilde{c}'\left(\theta\right) Q\left(p\left(\theta\right)\right) - \beta\left(\theta\right) \int_{\Gamma} N\left(\tilde{\theta}, C\right) \frac{\partial h\left(C|\theta\right)}{\partial\theta} dC \tag{9}$$

Constraints

Incentive-compatibility (truth-telling)

$$\pi(\theta) \geq \pi(\hat{\theta}, \theta) \qquad \forall \hat{\theta}, \theta \in [\theta_0, \theta_1],$$

participation

$$\pi(\theta) \geq 0, \qquad \forall \theta \in [\theta_0, \theta_1].$$

$$0 \leq \beta(\theta) \leq 1, \qquad \forall \theta \in [\theta_0, \theta_1] \tag{6}$$

$$0 \leq N(\theta, C) \leq \bar{N}, \qquad \forall \theta \in [\theta_0, \theta_1], \qquad \forall C \in \Gamma. \tag{7}$$

Lagrangian

$$L = \left\{ V\left(Q\left(p\left(\theta\right)\right)\right) - \tilde{c}\left(\theta\right) Q\left(p\left(\hat{\theta}\right)\right) - k - a\beta\left(\theta\right) - \left(1-\alpha\right)\pi\left(\theta\right) \right\} f\left(\theta\right)$$

$$+\mu\left(\theta\right)\left(-\tilde{c}'\left(\theta\right) Q\left(p\left(\theta\right)\right) - \beta\left(\theta\right)\int_{\Gamma} N\left(\tilde{\theta}, C\right)\frac{\partial h\left(C|\theta\right)}{\partial\theta} dC\right) + \tau\left(\theta\right)\pi\left(\theta\right)$$

"co-state" variable

Multiplier for individual rationality constraints

“Endpoint” Cases

Lemma 1. The costate variable $\mu(\theta)$ satisfies

$$\mu(\theta) = (1 - \alpha)F(\theta) - \int_{\theta_0}^{\theta} \tau(v)dv \qquad \forall \theta \in [\theta_0, \theta_1] \tag{10a}$$

and

$$\mu(\theta_0) = 0; \qquad \mu(\theta_1)\pi(\theta_1) = 0. \tag{10b}$$

Lemma 2. In an incentive-compatible policy the marginal cost of satisfying the local-incentive-compatibility condition in (9) is nonnegative, i.e., $\mu(\theta) \geq 0$ for all $\theta \in [\theta_0, \theta_1]$.

First Characterization of Policy: Stochastic Passage Rule

Proposition 1. Suppose the induced-likelihood function $h(C|\theta)$ is continuously differentiable and satisfies the monotone-likelihood ratio property. Let $\theta^*(C)$ be the (unique) maximum-likelihood estimator for the parameter θ when the cost realization is C. Suppose the maximum likelihood estimator is strictly increasing in C. Let $Z^*(\theta)$ denote the inverse of $\theta^*(C)$.[17] Then the optimal penalty function in the transformed problem is given by

$$N(\hat{\theta}, C) = \begin{cases} \bar{N} & \text{if} \quad C < Z(\hat{\theta}) \\ 0 & \text{if} \quad C \geq Z(\hat{\theta}). \end{cases}$$

Proposition 2. If $F(\theta)/f(\theta)$ is nondecreasing in θ, $\tilde{c}$ is normal with parameters $(\bar{c}(\theta), \sigma^2)$, and $\bar{c}(\theta)$ is a differentiable, nondecreasing, and (weakly) convex function of θ, the optimal solution, denoted by an $*$, to the transformed program is a set of functions given by

$$Z^*(\theta) = \bar{c}(\theta)Q(p^*(\theta)) + k = E(\tilde{C}|\theta) \tag{12}$$

$$E(N^*(\theta, \tilde{C})|\theta) = \frac{\bar{N}}{2} \tag{13}$$

$$\beta^*(\theta) = \begin{cases} 0 & \text{if} \quad \dfrac{\mu(\theta)\bar{N}\bar{c}'(\theta)}{\sqrt{2\pi}\sigma} - af(\theta) < 0 \\ 1 & \text{if} \quad \dfrac{\mu(\theta)\bar{N}\bar{c}'(\theta)}{\sqrt{2\pi}\sigma} - af(\theta) > 0 \\ \in[0, 1] & \text{if} \quad \dfrac{\mu(\theta)\bar{N}\bar{c}'(\theta)}{\sqrt{2\pi}\sigma} - af(\theta) = 0 \end{cases} \tag{14}$$

$$p^*(\theta) = \bar{c}(\theta) + \bar{c}'(\theta)\,\frac{\mu(\theta)}{f(\theta)} \tag{15}$$

with

$$\tau(\theta) \geq 0; \qquad \tau(\theta)\pi(\theta) = 0. \tag{16}$$

FIGURE 1

AUDITING AND PRICING POLICIES WHEN INDIVIDUAL RATIONING CONSTRAINT IS BINDING: NORMAL CASE

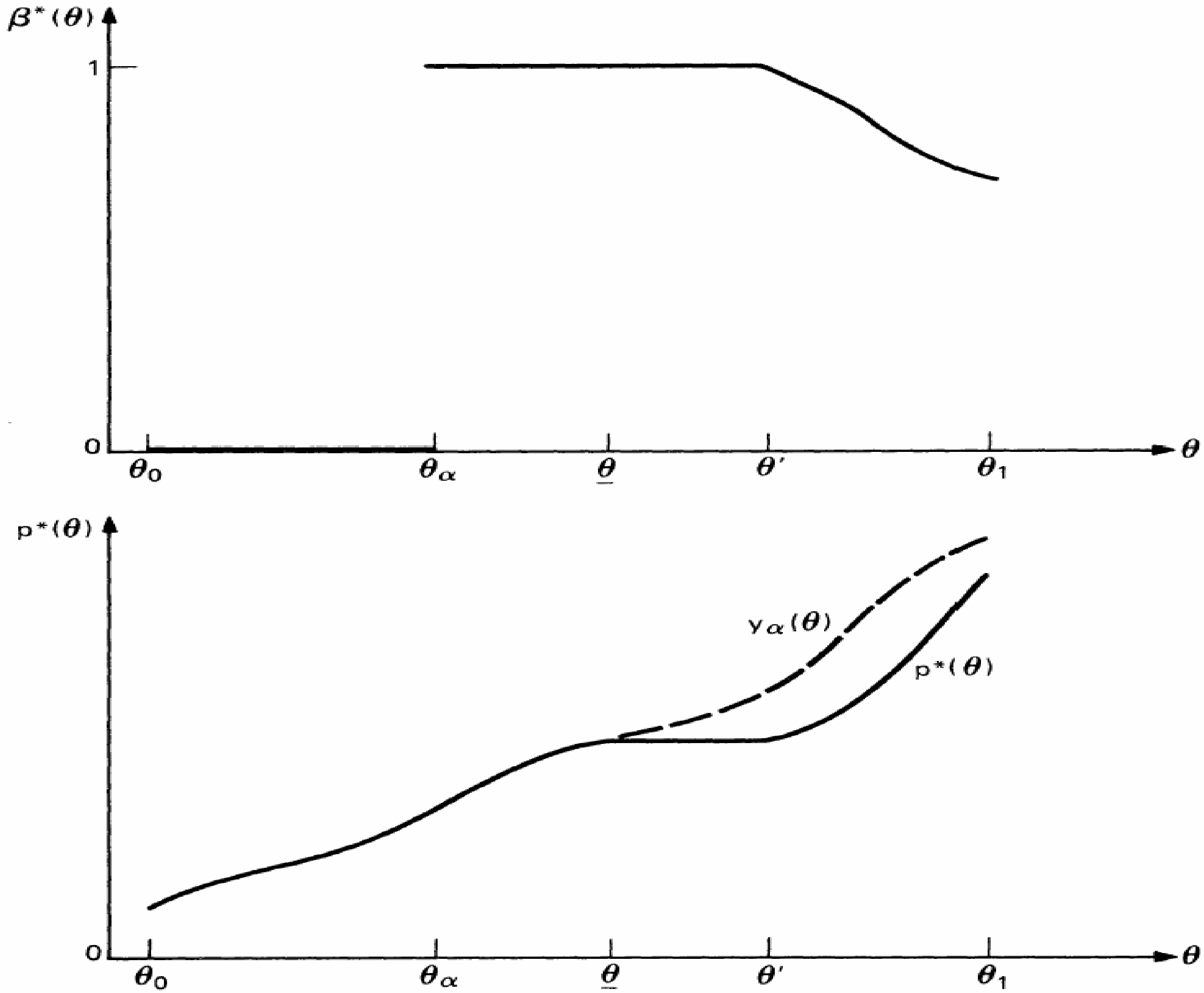

FIGURE 2

AUDITING REGION WITH SEPARATION